Exam. Code : 211003
Subject Code : 4861

M.Sc. Mathematics 3rd Semester

TOPOLOGY—I

Paper—MATH-572

Time Allowed—3 Hours] [Maximum Marks—100

Note :— Attempt ***five*** questions selecting at least ***one*** question from each section. The **fifth** question may be attempted from any section. All questions carry equal marks.

SECTION—A

1. (a) Show that a subset A of a topological space x is open if and only if Int(A) = A. 5

   (b) Let X = {a, b, c} and subsets of X are Ø, X. {a}, {b}, {c}, {a, b}, {a, c}, {b, c}. Let T be defined as {Ø, {a}, {a, b}, X}. Is T a topology on X ? Justify your answer. 5

2. Define Kuratowski closure operator and explain how it induces topology. 10

SECTION—B

3. (a) Let (X, T_1) and (Y, T_2) be two topological spaces, then a function $f : X \to Y$ is continuous if and only if $f(\overline{A}) \subset \overline{f(A)}$. 7

   (b) Show that a continuous map need not be an open map. 3

4. (a) Define locally connected space. Prove that a space is locally connected if and only if components of each open subset of X is open. 5

(b) Let (X, T) be a topological space and (Y, T_Y) be a subspace (X, T). State and prove necessary and sufficient condition for every T_Y-open set to be T-open. 5

SECTION—C

5. (a) Let $\Pi_{i=1}^{n} Y_i$ is a product space. Prove that $\forall\ A_i \subset Y_i\ \Pi_{i=1}^{n} A^{\circ} = \left(\Pi_{i=1}^{n} A_i\right)^{\circ}$. 5

(b) Prove that $\Pi_{\alpha\in\Lambda} Y_{\alpha}$ is connected if and only if each Y_{α} is connected. 5

6. (a) Show that :

$$Fr(A_1 \times A_2) = (Fr(A_1) \times \bar{A}_2) \cup (\bar{A}_1 \times Fr(A_2)).$$ 5

(b) Show that infinite product of discrete space is never discrete. 5

SECTION—D

7. State and prove Urysohn's lemma. 10

8. (a) Let (X, T) be a topological space. Prove that the following statements are equivalent :

(i) X is normal.

(ii) If $A \subset U$ and A is closed and U is open then there exist open set V such that

$$A \subset V \subset \bar{V} \subset U.$$

(iii) If A and B are disjoint closed sets then there exist an open set U containing A such that $\bar{U} \cap B = \emptyset$. 6

(b) Prove that regularity is a hereditary property. 4